International Journal of Scientific & Engineering Research, Volume 6, Issue 2, February-2015
ISSN 2229-5518


# Application of Fuzzy Graph in Traffic

R.Myna,

**Abstract—** In this paper, we use a fuzzy graph model to represent a traffic network of a city and discuss a method to find the different type of accidental zones in a traffic flows using Edge coloring of a fuzzy graph.

**Index Terms**— connectivity, fuzzy cut nodes, fuzzy bridges, coloring of a graph, arcs in a fuzzy graph , traffic lights.

## 1 Introduction

Fuzzy graphs were introduced by Rosenfeld, ten years after Zadeh's landmark paper ''Fuzzy Sets". Rosenfeld has obtained the fuzzy analogues of several basic graph-theoretic concepts like bridges, paths, cycles, trees and connectedness and established some of their properties.

Bhattacharya has established some connectivity concepts regarding fuzzy cut nodes and fuzzy bridges. Graph coloring is one of the most important concepts in graph theory and is used in many real time applications like job scheduling, aircraft scheduling, computer network security, map coloring and GSM mobile phone networks, automatic channel allocation for small wireless local area networks. The proper coloring of a graph is the coloring of the vertices with minimal number of colors such that no two adjacent vertices should have the same color. The minimum number of colors is called as the chromatic number and the graph is called properly colored graph .

## 2 Preliminary Notes

In this Section, we present some definitions related to fuzzy graph and their properties.

**Definition 2.1.** Fuzziness occurs in a fuzzy graph in five different ways, introduced by M.Blue. Fuzzy graph is a graph GF satisfying one of the following types of fuzziness (GF of the ith type) or any of its combination:

(i) $GF_1=\{G_1, G_2, G_3, \ldots, GF\}$ where fuzziness is on each graph Gi

(ii) $GF_2=\{V, EF\}$ where the edge set is fuzzy.

(iii) $GF_3=\{V, E(tF, hF)\}$ where both the vertex and edge sets are crisp, but the edges have fuzzy heads $h(e_i)$ and fuzzy tails $t(e_i)$

(iv) $GF_4=\{V_F, E\}$ where the vertex set is fuzzy.

(v) $GF_5=\{V, E(w_F)\}$ where both the vertex and edge sets are crisp but the edges have fuzzy weights.

In this paper, we use a fuzzy graph G which is a combination of $GF_2$ and $GF_4$. So fuzzy graph $G= GF_2 \cup GF_4$ . We can define this fuzzy graph using the membership values of vertices and edges. Let *V* be a finite nonempty set. The triple $G= (V,E, \sigma, \mu)$ is called a fuzzy graph on V where $\mu$ and $\sigma$ are fuzzy sets on V and E (V × V) respectively, such that $\mu(\{u, v\}) \le \min\{\sigma(u), \sigma(v)\}$ for all $u, v \in V$.

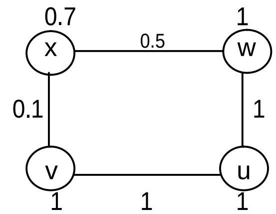
Let us consider a fuzzy graph $G : (V,E, \sigma, \mu)$ be $V=(u,v,w,x)$, $E=\{uv,uw,wx,xv\}$ where $\mu(u,v)=1= \mu(u,w)$, $\mu(x,w)=0.5$, $\mu(v,x)=0.1$ and $\sigma=\{1,1,1,0.7\}$.

Note that a fuzzy graph is a generalization of crisp graph in which

$\mu(v) = 1$ for all $v \in V$ and

$\rho(i, j) = 1$ if $(i, j) \in E$

$= 0$ otherwise




- *R.Myna is currently pursuing masters degree program in Mathematics in Bharathiar University, TamilNadu, India, E-mail: mynagopal@gmail.com*

**Definition 2.2.** The $\alpha$ cut of fuzzy graph defined as $G_\alpha = (V_\alpha, E_\alpha, \sigma, \mu)$ where $V_\alpha = \{v \in V \mid \sigma \geq \alpha\}$ and $E_\alpha = \{e \in E \mid \mu \geq \alpha\}$.

**Definition 2.3.** A path P of length n is a sequence of distinct nodes $u_1, u_2, u_3, \ldots u_n$ such that $\mu(u_{(i-1)} u_i) > 0$; $i = 1, 2, \ldots, n$ and the degree of membership of a weakest arc is defined as its strength. The strength of the path is defined as $\min(\mu(u_{(i-1)} u_i))$.

**Definition 2.4.** A fuzzy graph $G=(V,E,\sigma,\mu)$ is called strong if $\mu(u,v) = \min(\sigma(u), \sigma(v))$ for all $(u,v)$ in $\mu^*$ and is complete if $\mu(u,v) = \min(\sigma(u), \sigma(v))$ for all $(u,v)$ in $\sigma^*$.

**Definition 2.5.** The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x,y)$. An x- y path P is called a strongest x- y path if its strength equals to $CONN_G(x,y)$

## 3 Types of arcs in a fuzzy graph

The notion of strength of connectedness plays a significant role in the structure of fuzzy graph. Depending on the value $CONN_G(x,y)$ of an arc $(x,y)$ in a fuzzy graph G we define the following three different types of arcs. Note that $CONN_{G-(x,y)}(x,y)$ is the strength of connectedness between x and y in the fuzzy graph obtained from G by deleting the arc(x,y).

**Definition 3.1.** An arc (x,y) of G is called $\alpha$-strong if $\mu(xy) > CONN_{G-(x,y)}(x,y)$.

**Definition 3.2.** An arc (x,y) of G is called $\beta$-strong if $\mu(xy) = CONN_{G-(x,y)}(x,y)$

**Definition 3.3.** An arc (x,y) of G is called $\delta$-strong if $\mu(xy) < CONN_{G-(x,y)}(x,y)$

**Definition 3.4.** An arc (x,y) of G is called $\delta^*$-strong if $\mu(xy) > \mu(uv)$ where (u,v) is the weakest arc in fuzzy graph

**Definition 3.5.** An fuzzy graph $G=(V,E,\sigma,\mu)$ is called an $\alpha$-strong path if all its arcs are $\alpha$ strong and is called a $\beta$ -strong path if all its arcs are $\beta$-strong.

## 4 Representing the traffic lights problem using fuzzy graph

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. In many real world problems, we get partial information about that problem. So there is vagueness in the description of the objects or in its relationships or in both. To describe this type of relation, we need to design graph model with fission of type 1 fuzzy set. This fission of fuzzy set with graph is known as fuzzy graph.

In this paper, we represent the traffic flows as a fuzzy graph problem. Let we consider a traffic flow shown in Fig 4. 1. Each arrow in Fig 4. 1 indicates the vehicles will go from one direction to another direction. But numbers of vehicles in all paths are not always equal. Due to this reason, we consider it as fuzzy set whose membership value depends upon on vehicles number. If the number of vehicles in any path is high then its membership value will be high and if the number of vehicles in any path is low then its membership value will be low.

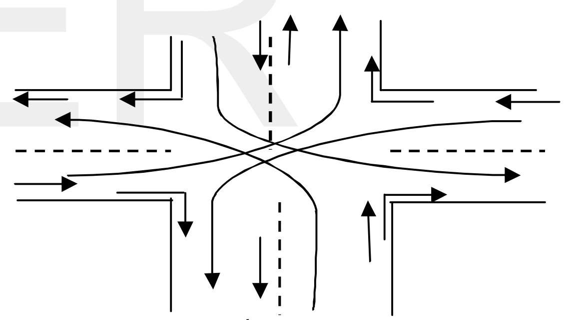


Fig 4.1

Since the four right turns do not interfere with the other traffic flows, they can safely be dropped from our discussion. The remaining traffic directions are shown in figure 5.2 and are labeled A through H and their membership values are depicted in table1.If the number of vehicles in any path is greater than 10000 per hour than we consider the membership value of that path is high. If the number of vehicles in any path is greater than or equal to 5000 per hour than we consider the membership value of that path is medium. If the number of vehicles in any path is less than 5000 per hour than we consider the membership value of that path is low. Membership values are represented by symbolic name H for high, M for medium, L for low respectively.

Table 4.1: Membership values of the vertices

| Vertex | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| $\Sigma$ | M | H | M | L | M | H | M | L |

We need to develop a traffic pattern so that vehicles can pass through the intersection without interfering with other traffic flows.

In this problem, we represent each traffic flow using the vertices of the fuzzy graph and their membership value depends upon the number of the vehicle of that road. Two vertices are adjacent if the corresponding traffic flows cross each other. For instance, direction C and H intersect, so vertices C and H are adjacent. If two vertices are adjacent then there is a possibility of accident. The possibility of accident depends on the adjacent vertices membership value which represents number of vehicles on the road.

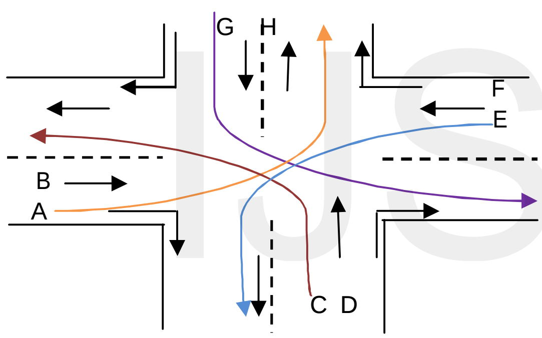



Fig 4.2

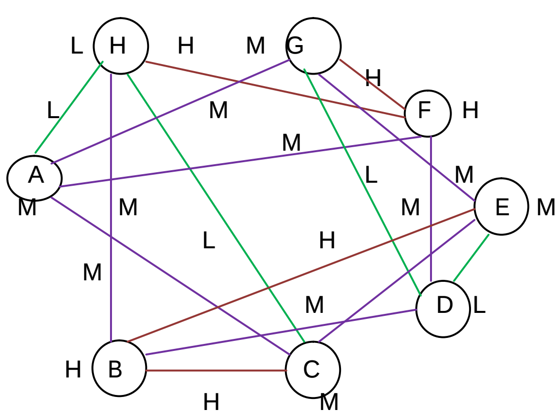



Fig 4.3

If membership values of the adjacent two vertices are high (H) then we consider the membership value of that arc is high (H). We consider the arc membership value is low (L) if both the vertices have low (L) membership or one adjacent vertex has low (L) membership value and another node has medium(M) membership value. In another condition, if the membership value of one adjacent vertex is high (H) and another has low (L) membership value or both node has medium (M) membership value then we consider the membership value of that arc is medium (M).

In this paper, we represent each possibility of accident with an edge and their membership value. Membership values of edges are given below in Table 2.

Table 4.2 : Membership values of the edges

| Edge | AH | AG | AF | AC | BC | BD | BH | BE |
|---|---|---|---|---|---|---|---|---|
| $\mu$ | L | M | H | M | H | M | M | H |
| Edge | CH | DE | DF | DG | GE | FG | FH | CE |
| M | L | L | M | L | M | H | H | M |

## 5 Types of accidental zone in a traffic network

Depending upon the membership values of the edges and $CONN_G$ (x,y) of an arc (x,y) in a fuzzy graph G , we can classify the different type of accident zones in traffic flow. We define three different type of accidental zone in the traffic flow. Consider a fuzzy graph $G=(V,E\ \sigma,\mu)$. Using the fuzzy graph (G), we represent a traffic flow of a city. Let u,v be two routes in the traffic flows and two vertices are adjacent if the corresponding traffic flows cross each other.

Let e=(u,v) be an arc in graph G such that $\mu(u,v)=x>0$. Then do the following step:

1. Obtain G- e
2. Find the value of $CONN_{(G-e)}$.
3. (a) If x > $CONN_{(G-e)}$ then e is $\alpha$-strong accidental zone.

(b) If $CONN_{(G-e)}$ =x then e is $\beta$-strong accidental zone.

(c) If $CONN_{(G-e)}$ >x then e is $\delta$-strong accidental zone.

4. Repeat steps 1–4 for all arcs in G.

We can apply this method to classify the accidental zone of a traffic flows in a city. We apply the method to our problem (Fig 5.3). In this fuzzy graph $\alpha$-strong accidental zone are AF,BC,BE,FG and $\beta$-strong accidental zone are AG,AC,BD,BH,DF,GE,CE and $\delta$-strong accidental zone are AH, CH, DE, DG. This classification will help control the traffic flow of a city.

## 6 Coloring the fuzzy graph

We color this fuzzy graph using the concept of Eslahchi and Onagh. They defined fuzzy chromatic number as the least value of k for which the fuzzy graph G has k –fuzzy coloring and k –fuzzy coloring is defined as follows. A family $\Gamma=\{\gamma_1, \gamma_2 \ldots \gamma_k\}$ of fuzzy sets on V is called a k -coloring of fuzzy graph $G=(V, E, \sigma, \mu)$

(a) $\vee \Gamma = \sigma$,

(b) $\gamma_1 \cap \gamma_2 = 0$

(c) for every strong edge xy of G ,min $\{\gamma_1(x), \gamma_2(y)\}=0$ $(1 \le i \le k)$.

Let $\Gamma=\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ be a family of fuzzy sets defined on V by

$$\gamma_1(v_i)=\begin{cases} M & \text{if } i=A \\ M & \text{if } i=E \\ 0 & \text{otherwise} \end{cases} \qquad \gamma_2(v_i)=\begin{cases} H & \text{if } i=B \\ H & \text{if } i=F \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma_3(v_i)=\begin{cases} L & \text{if } i=A \\ L & \text{if } i=H \\ 0 & \text{otherwise} \end{cases} \qquad \gamma_4(v_i)=\begin{cases} M & \text{if } i=C \\ M & \text{if } i=G \\ 0 & \text{otherwise} \end{cases}$$

Here we can see that the family $\Gamma=\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ satisfies the conditions of definition of vertex coloring. Thus the fuzzy graph in our example have 4-fuzzy vertex coloring and this is the minimal vertex fuzzy coloring since any family with less than 4 members does not satisfy the conditions of the definition. Thus the fuzzy vertex chromatic number $\chi(G) = 4$.

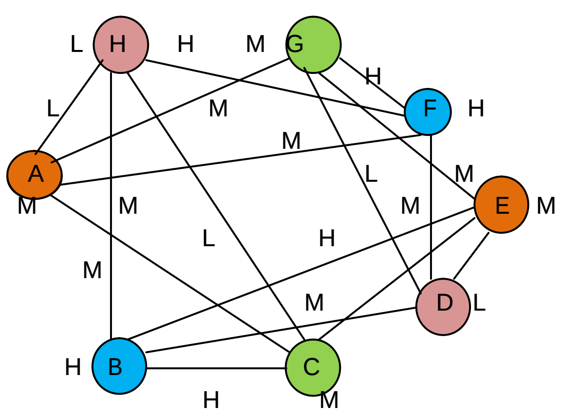



Fig 6.1

Fig 6.1 shows a coloring of graph with exactly four colors, which depicts an
efficient way of designing the traffic signal pattern. It consists of four phases:

Table 6.1: Traffic light control pattern

| Traffic light pattern | | | |
|---|---|---|---|
| Phase 1 | Phase 2 | Phase 3 | Phase 4 |
| Only B and F proceed | Only D and H proceed | Only A and E proceed | Only C and G proceed |

In our traditional road traffic lights have the same cycle time T. So the duration of the all the phase will be equal and that is T. In this problem, we assume that number of vehicles in B and F direction is maximum and D and H direction is minimum. That means number of vehicles in all paths are not equal. If D and H need T time to pass all the vehicles then B and F will need more time than T. So total waiting time of vehicles on the roads will be increase and there may be a possibility of traffic jam or accident. Using fuzzy graph, we can solve this problem. We can give the duration time of the traffic light depends upon on the vehicle number (it is represented by vertex membership value). If the node membership value is high then it needs more time to flow the all vehicles. In this problem duration of the Phase 2 will be maximum and Phase 3 will be minimum. Using this concept, total waiting time of the vehicles will be minimizing.

## 7 Conclusion

In the concept of crisp incompatibly among the nodes of graph we cannot describe the
vagueness or partial information about a problem. In this paper, we represent the traffic
flows using a fuzzy graph whose vertices and edges both are fuzzy vertices and fuzzy
arcs. So we can describe vagueness in vertices and also in edges. Using this Membership
value of edges and vertices, we have introduced a method to classify the accidental zone of a traffic flows. We can give a speed limit of vehicle according to accidental zone. The chromatic number of G is $\chi(G) = \{(4, L), (3,M),(1,H)\}$. The interpretation of $\chi(G)$ Is the following: lower values of $\alpha$ are associated to lower driver aptitude levels and, consequently, the traffic lights must be controlled conservatively and the chromatic number is high; on the other hand, for higher values of $\alpha$, the driver aptitude levels
increase and the chromatic number is lower, allowing a less conservative control of the
traffic lights and a more fluid traffic flow.

## REFERENCES

[1] Roberts FS. On the mobile radio frequency assignment problem and the traffic light phasing problem. Annals of the New York Academy of Sciences 1979; 319: 466-83.
[2] Zadeh LA. Similarity relations and fuzzy ordering. Information Sciences 1971;3(2):177-200.
[3] Rosenfeld A. Fuzzy graphs. In: Zadeh LA, Fu KS, Shimura M, editors. Fuzzy sets and their applications to cognitive and decision processes. New York: Academic Press; 1979 p. 77-95.
[4] Golumbic MC. Algorithmic graph theory and perfect graphs. New York: Academic Press; 1980.
[5] Opsut RJ, Roberts FS. I-colorings, I- phasings, and I-intersection assignments for graphs, and their applications. Networks 1983; 13:327-45.
[6] Stoffers KE. Scheduling of traffic lights. A new approach. Transportation Research 1968; 2: 199-234